Name [illegible], ID [illegible]

Exam II: MTH 111, Spring 2017

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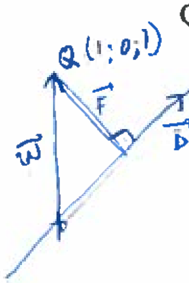
Points = $\frac{41}{42}$ Excellent!!

QUESTION 1. (4 points) The point $Q = (1, 0, 1)$ is not on the line $L: x = 2t - 1, y = 2, z = 2t - 1$. Find $|QL|$



• DIRECTION VECTOR OF L: $\vec{D} \langle 2; 0; 2 \rangle$

• INITIAL POINT : for $t=0$, $I = (-1; 2; -1)$

$\vec{D} \cdot \vec{W} = \langle 1+1; 0-2; 1+1 \rangle = \langle 2; -2; 2 \rangle$

$L: x = 2t - 1$
$y = 2$
$z = 2t - 1$

• WE FIND $\text{proj}_{\vec{D}} \vec{W} = \frac{\vec{W} \cdot \vec{D}}{|D|^2} \times \vec{D} = \frac{8}{8} \times \langle 2;0;2 \rangle = \langle 2;0;2 \rangle$

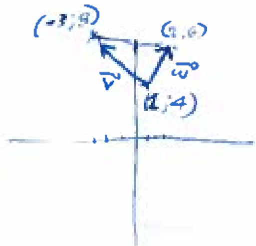
$\rightarrow F = \vec{W} - \text{proj}_D W = \langle 2;-2;2 \rangle - \langle 2;0;2 \rangle = \langle 0;-2;0 \rangle$

$|F| = |QL| = \boxed{2}$

QUESTION 2. (3 points) The point $Q = (1, 1, 1)$ is not on the plane $P: 2x + 2y + z - 11 = 0$. Find $|QP|$.

$|QP| = \frac{|2(1) + 2(1) + (1) - 11|}{|\sqrt{2^2 + 2^2 + 1^2}|} = \frac{6}{3} = \boxed{2}$

QUESTION 3. (3 points) Find the area of the triangle with the following three vertices: $(1, 4), (2, 6), (-3, 8)$



$\vec{N} = \langle -3-1; 8-4 \rangle = \langle -4; 4 \rangle$

$\vec{W} = \langle 2-1; 6-4 \rangle = \langle 1; 2 \rangle$

WE PUT THE VECTORS IN THE THIRD DIMENSION.

$|V \times W| = \begin{vmatrix} i & j & k \\ -4 & 4 & 0 \\ 1 & 2 & 0 \end{vmatrix} = \begin{vmatrix} 4 & 0 \\ 2 & 0 \end{vmatrix} i - \begin{vmatrix} -4 & 0 \\ 1 & 0 \end{vmatrix} j + \begin{vmatrix} -4 & 4 \\ 1 & 2 \end{vmatrix} k$

$= 0i - 0j + (-12)k = \langle 0; 0; 12 \rangle$

$|V \times W| = 12$ area $= \frac{12}{2} = 6$ UNIT^2.

QUESTION 4. (6 points) Given $f(x)$ is a function such that $f'(x)$ is defined on all real numbers. Given that $f'(x) = 0$ only when $x = -4$ and $x = 4$. The equation of the normal line to the curve of $f(x)$ when $x = 0$ is $y = 3x + 2$. The equation of the tangent line to the curve of $f(x)$ when $x = -6$ is $y = -4x + 3$. The equation of the normal line to the curve of f(x) when $x = 7$ is $y = -7x + 11$.

(i) For what values of x does $f(x)$ decrease?

slope tangent positive : $f(x) \nearrow$

$f'(x) > 0$: $f(x) \nearrow$

if slope normal line at $x=0$ is 3, the slope of tangent $= -\frac{1}{3} < 0$ so $f(x)$ decreasing. $(f'(x) < 0)$

$\ominus$ -4 $\ominus$ 4 $\oplus$

$f(x)$ is decreasing for $x \in (-\infty; 4]$

(ii) For what values of x does $f(x)$ increase?

$f(x)$ increases for $x \in [4; +\infty)$

(iii) For what values of x does $f(x)$ have a local minimum value?

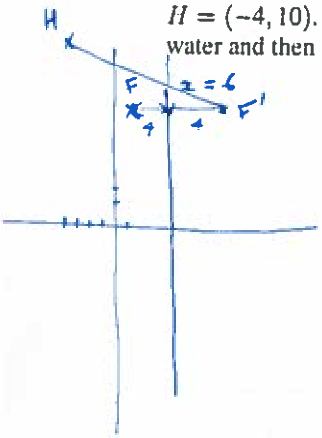
$f(x)$ has a local minimum value for $x = 4$;

(iv) For what values of x does $f(x)$ have a local maximum value?

according to the sign of $f(x)$, $f(x)$ does not have a local maximum.

$\ominus$ -4 $\ominus$ 4 $\oplus$

QUESTION 5. (5 points) There is a fire-station located at the point $F = (2, 8)$. A house is on fire and it is located at $H = (-4, 10)$. There is a river that is located at $x = 6$. The fire-men want to find a point Q on the river in order to get water and then travel to the House such that $|FQ| + |QH|$ is minimum. Find Q.



~~WE FLIP F ABOUT $x = 6$~~

~~$F' = (10; 8)$ AT EQUAL DISTANCE TO $x = 6$ THAN F. (4 UNIT)~~

~~HF' POSSESSES THE FOLLOWING EQUATION:~~

~~$y = mx + b$; $m = \frac{\Delta y}{\Delta x} = \frac{10+4}{8-10} = -7$~~

~~$10 = -7(-4) + b$, $b = -18$~~

~~$\boxed{y = -7x - 18}$ NOW WE NEED TO FIND Q: INTERSECTION pt. BETWEEN [HF'] and $x = 6$, THUS $Q = (6; 36)$~~

PLEASE TURN OVER

QUESTION 6. (4 points) Find the equation of the tangent line to the curve of $f(x) = 12\sqrt{x} - 5x + 1$ at the point $(4, 5)$.

$f(x) = 12 \times x^{1/2} - 5x + 1$

$f'(x) = 12 \times \frac{1}{2} x^{-1/2} - 5 = 6x^{-\frac{1}{2}} - 5$

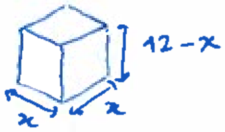
$x = 4$; $f'(4) = 6(4)^{-1/2} - 5 = -2$

THUS ; in $y = mx + b$, $m = -2$.

$5 = -2(4) + b$; $b = 13$

EQUATION OF TANGENT LINE AT $(4; 5)$ is AS FOLLOWS: $y = -2x + 13$.

QUESTION 7. (5 points) Imagine that you want to construct a box that has a square base, say of length x (and hence it has width x), and with height $12 - x$ so that the volume is maximum. What is the value of x? (note that Volume = length X width X Height)



VOLUME $= L \times W \times H$

$= x \times x \times (12 - x) = x^2(12 - x)$

$V = x^2(12 - x)$

$V' = 2x(12 - x) + x^2(-1) = 24x - 2x^2 - x^2$

$= 24x - 3x^2$

$V' = 0$; $24x - 3x^2 = 0$

$x(24 - 3x) = 0$

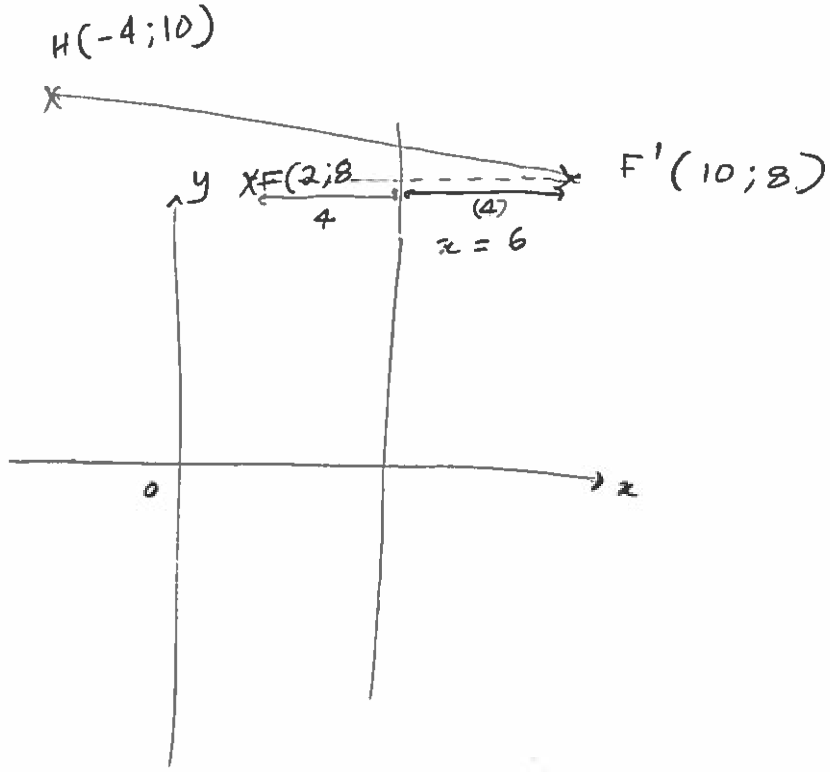
$x = 0$ CANCELED $(x \neq 0)$

OR $x = 24$

$V'' = 24 - 2x$

$V''(24) = -24 < 0$ so V is MAXIMAL at $x = 24$.

QUESTION 5



F (2;8)

H (-4;10)

$x = 6$.

H (-4;10)

F' (10;8)

$y = mx + b$

$$m = \frac{\Delta y}{\Delta x} = \frac{10-8}{-4-10} = \frac{2}{-14} = -\frac{1}{7}$$

$$10 = -\frac{1}{7}(-4) + b$$

$$10 = \frac{4}{7} + b \qquad b = \boxed{\frac{66}{7}}$$

$$y = -\frac{1}{7}x + \frac{66}{7}$$

COORDINATES OF Q : $y = -\frac{1}{7} \times 6 + \frac{66}{7}$

$$= \frac{60}{7}$$

$Q = \left(6 ; \frac{60}{7}\right)$

FOR $|FQ| + |QH|$ TO BE MINIMAL.

QUESTION 8. (6 points) Let $f(x) = -e^x + e^{10}x + 4$

(i) For what values of x does $f(x)$ increase?

$f(x)$ INCREASES if $f'(x) > 0$.

$\left[a\left(b^{E(x)}\right)\right]' = a\left(b^{E(x)}\right) \times E'_x \times \ln$

$f(x) = -e^x + e^{10}x + 4$

$f'(x) = -e^x \times (1) \times (1) + \left[(e^{10} \times 0 \times 1)(x) + (1)(e^{10})\right] + 0$

$= -e^x + e^{10}$

$f'(x) = 0 \quad ; \quad -e^x + e^{10} = 0$

$-e^x = -e^{10}$

$\ln e^x = \ln e^{10}$

$x = 10$

$\oplus \quad 10 \quad \ominus$ sign of $f'(x)$

$f(x)$ increases for $x \in (-\infty ; 10)$.

(ii) For what values of x does $f(x)$ decrease?

for $x \in (10 ; +\infty)$, $f(x)$ decreases.

(iii) For what values of x does $f(x)$ have a maximum value?

looking at the sketch, $f(x)$ clearly has a maximum at $x = 10$;

~~f(10)~~ =

QUESTION 9. (6 points) Find y' and do not simplify

(i) $y = ln\left[\frac{(x+2)^3}{3x+7}\right]$ $= 3\ln(x+2) - \ln(3x+7)$

$\left[a \log_b(k(x))\right]' = \frac{a}{\ln(b)} \times \frac{k'(x)}{k(x)}$

$y' = \frac{3}{\ln(e)} \times \frac{1}{x+2} - \frac{1}{1} \times \frac{3}{3x+7}$

$\boxed{y' = \frac{3}{x+2} - \frac{3}{3x+7}}$

(ii) $y = \underbrace{(7x+3)}_{1}\underbrace{e^{(2x^2-5x)}}_{2} + 10x$

(1) $7x+3$

(1)' 7

(2) $e^{(2x^2-5x)}$

(2)' $e^{(2x^2-5x)} \times (4x-5)$

$y' = 7e^{(2x^2-5x)} + (7x+3)\left(e^{(2x^2-5x)} \cdot (4x-5)\right) + 10$

(iii) $y = ln\left((6x+2)^3(-7x+4)^7\right)$ $= 3\ln(6x+2) + 7\ln(-7x+4)$

$y' = \frac{3}{1} \times \frac{6}{6x+2} + \frac{7}{1} \times \frac{-7}{-7x+4}$ $\boxed{= \frac{18}{(6x+2)} + \frac{-49}{(-7x+4)}}$

Faculty information

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